MAGNETIC PROPERTIES OF MATTER

Outline:

- Some basic ideas related to magnetism.
- Dia, Para, Ferri and Ferromagnetic Materials.
- Classical Langevin Theory of Dia and Paramagnetic Domains.
- Quantum Mechanical Treatment of Paramagnetism (using parition function). Curie's law.
- Weiss's Theory of Ferromagnetism
- Ferromagnetic Domains. Discussion of B-H Curve. Hysteresis and Energy Loss.

Magnetic Materials: In simple words, those materials that interact with the magnetic field.

- **Soft Magnetic Material:** Does not have permanent magnetism and loses its magnetism quickly when the magnetic field is withdrawn. They have high permeability, low coercive field and low magnetic losses. Useful for magnetic cores of transformers, motors, inductors and generators.

- **Hard Magnetic Material:** Have permanent magnetism. They have high coercive field and a high residual magnetization.

Magnetic induction and magnetization:

When a magnetic field, ***H***, is applied to a material, the response of the material is called its *magnetic induction*, ***B***. The relationship between ***B*** and ***H*** is a property of the material. In some materials (and in free space), ***B*** is a linear function of ***H***, but in general it is much more complicated, and sometimes it's not even single-valued. The equation relating ***B*** and ***H*** is (in cgs units)

$$B = H + 4\pi M$$

Where, ***M*** is the *magnetization* of the medium. The magnetization is defined as the magnetic moment per unit volume,

$$M = m/V, \ (emu/cm^3)$$

M **is a property of the material, and depends on both the individual magnetic moments of the constituent ions, atoms, or molecules, and on how these dipole moments interact with each other. The cgs unit of magnetization is the emu/cm^3.**

One might expect that, since $B = H$ in free space (where $M = 0$), the unit of magnetic induction should be the same as that of magnetic field, that is, the oersted. In fact this is not the case, and in fact the unit of magnetic induction is called the gauss. Indeed, mixing up gauss and oersteds is a sure way to upset magnetism scientists at parties. If you have trouble remembering which is which, it can be safer to work in the SI units which we discuss next.

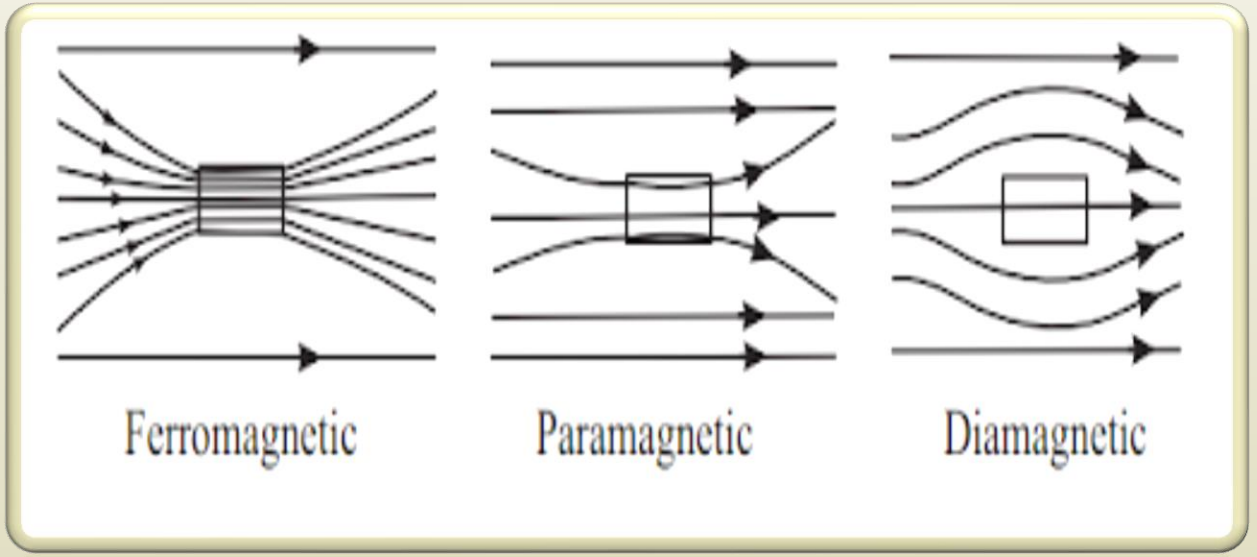
In SI units the relationship between *B*, *H*, and *M* is

$$B = \mu_0(H + M),$$

where μ_0 is the permeability of free space. The units of *M* are obviously the same as those of *H* (A/m), and those of μ_0 are weber/(A.m), also known as henry/m. So the units of *B* are weber/m^2, or tesla (T);

$$1 \text{ gauss} = 10^{-4} \text{ tesla}.$$

Flux density: The magnetic induction, *B*, is the same thing as the density of flux, Φ, *inside* the medium. So within a material $B = \Phi/A$, by analogy with $H = \Phi/A$ in free space. In general the flux density inside a material is different from that outside. In fact magnetic materials can be classified according to the difference between their internal and external flux. If Φ inside is *less than Φ* outside then the material is known as ***diamagnetic.*** If *Φ* inside is *slightly more* than *Φ* outside then the material is either ***paramagnetic or antiferromagnetic.*** Finally, if *Φ* inside is *very much greater* than *Φ* outside then the material is either ***ferromagnetic or ferrimagnetic***.



Susceptibility and permeability: The properties of a material are defined not only by the magnetization, or the magnetic induction, but by the way in which these quantities *vary* with the applied magnetic field.

The ratio of *M* to *H* is called the susceptibility:

$$\chi = M/H \ (\text{emu/cm}^3\ \text{Oe})$$

The susceptibility indicates how responsive a material is to an applied magnetic field. (Sometimes the symbol κ is used for the susceptibility per unit volume; then $\chi = \kappa/\rho$ emu/(g Oe) is the susceptibility per unit mass.)The ratio of *B* to *H* is called the permeability:

$$\mu = B/H \ \ (\text{gauss/Oe})$$

μ indicates how *permeable* the material is to the magnetic field. A material which concentrates a large amount of flux density in its interior has a high permeability. Using the relationship $B = H + 4\pi M$ gives us the relationship (in cgs units) between permeability and susceptibility:

$$\mu = 1 + 4\pi\chi.$$

In SI units:

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

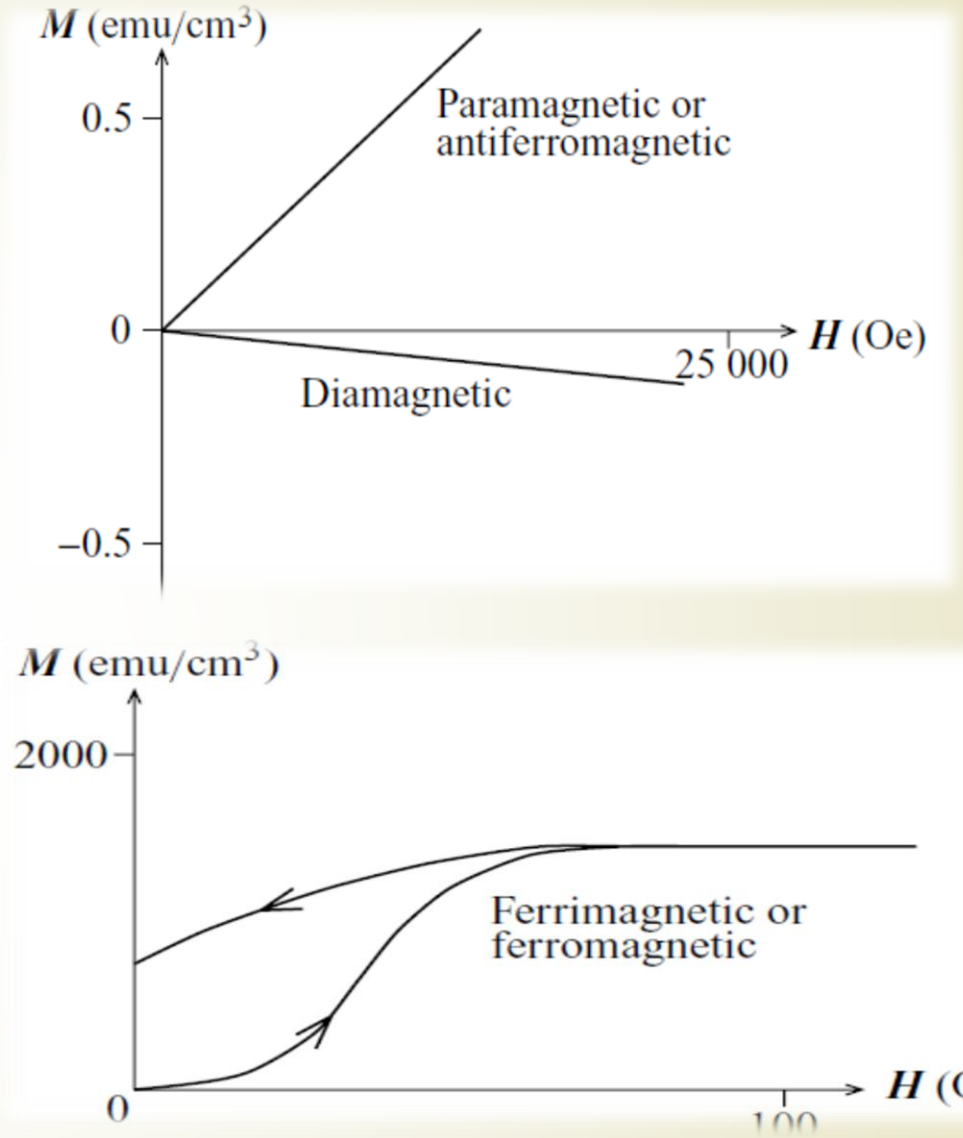
$$= \mu_0 \mathbf{H} (1 + \chi)$$

$$\mu \mathbf{H} = \mu_0 \mathbf{H} (1 + \chi)$$

$$\frac{\mu}{\mu_0} = (1 + \chi)$$

Note that in SI units the susceptibility is dimensionless, and the permeability is in units of henry/m. The corresponding relationship between permeability and susceptibility in SI units is

$$\frac{\mu}{\mu_0} = (1 + \chi)$$



Schematic magnetization curves for diamagnetic, paramagnetic, and antiferromagnetic materials.

Some Definitions

- ✓**Magnetic induction**, B. The magnetic induction is the response of a material to a magnetic field, H.
- ✓**Magnetization**, M. The magnetization is the total magnetic moment per unit volume.
- ✓**Susceptibility**, χ. The susceptibility is the ratio of M to H.
- ✓**Permeability**, μ. The permeability is the ratio of B to H.

Units related to Magenetism and conversions

	cgs	SI
Force between poles	$F = \frac{p_1 p_2}{r^2}$ (dyne)	$F = \frac{\mu_0}{4\pi}\frac{p_1 p_2}{r^2}$ (newton)
Field of a pole	$\boldsymbol{H} = \frac{p}{r^2}$ (oersted)	$\boldsymbol{H} = \frac{p}{r^2}$ (ampere/m)
Magnetic induction	$\boldsymbol{B} = \boldsymbol{H} + 4\pi\boldsymbol{M}$ (gauss)	$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ (tesla)
Energy of a dipole	$E = -\boldsymbol{m} \cdot \boldsymbol{H}$ (erg)	$E = -\mu_0 \boldsymbol{m} \cdot \boldsymbol{H}$ (joule)
Susceptibility	$\chi = \frac{\boldsymbol{M}}{\boldsymbol{H}}$ (emu/(cm^3 oersted))	$\chi = \frac{\boldsymbol{M}}{\boldsymbol{H}}$ (dimensionless)
Permeability	$\mu = \frac{\boldsymbol{B}}{\boldsymbol{H}} = 1 + 4\pi\chi \left(\frac{\text{gauss}}{\text{oersted}}\right)$	$\mu = \frac{\boldsymbol{B}}{\boldsymbol{H}} = \mu_0(1+\chi) \left(\frac{\text{henry}}{\text{m}}\right)$

Unit conversions

F	1 dyne	=	10^{-5} newton
$\boldsymbol{H}$	1 oersted	=	79.58 ampere/m
$\boldsymbol{B}$	1 gauss	=	10^{-4} tesla
E	1 erg	=	10^{-7} joule
Φ	1 maxwell	=	10^{-8} weber
$\boldsymbol{M}$	1 emu/cm^3	=	12.57×10^{-4} weber/m^2
μ	1 gauss/oersted	=	1.257×10^{-6} henry/m

It is often useful to convert the SI units into their fundamental constituents, ampere (A), meter (m), kilogram (kg), and second (s). Here are some examples.

newton (N)	=	kg m/s^2
joule (J)	=	kg m^2/s^2
tesla (T)	=	kg/(s^2 A)
weber (Wb)	=	kg m^2/(s^2 A)
henry (H)	=	kg m^2/(s^2 A^2)

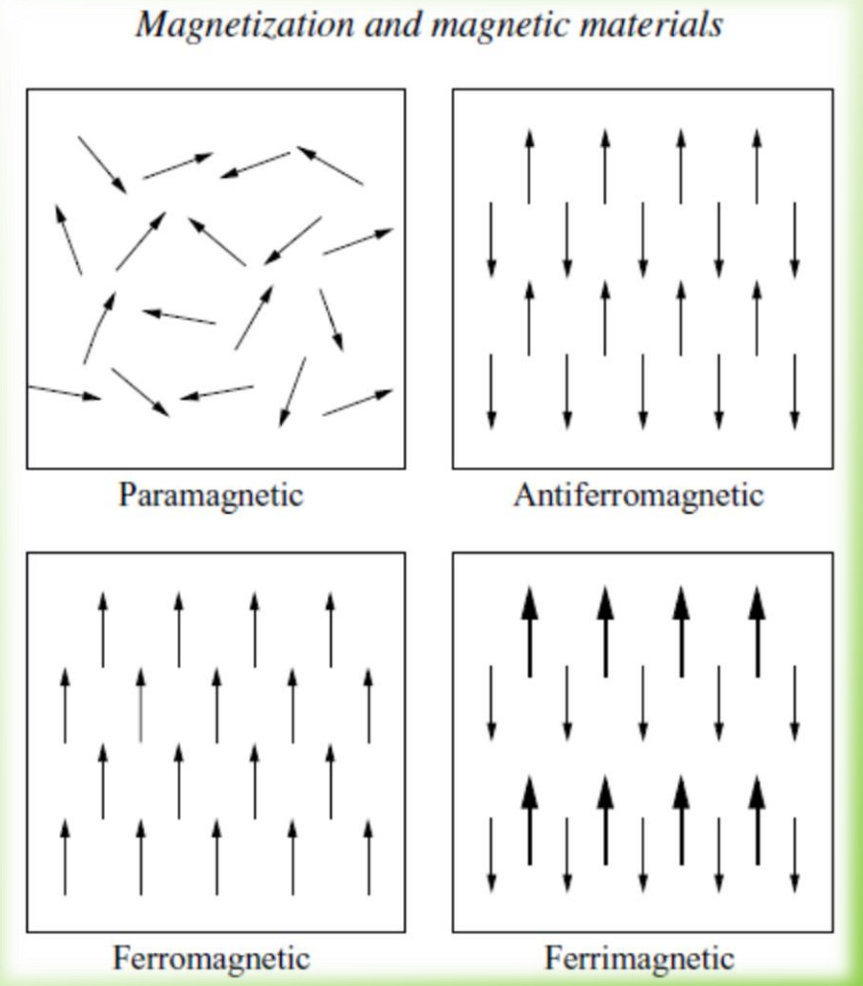
Magnetic properties of solids are grouped under the following headings:

I. Diamagnetism

II. Paramagnetism

III. ferromagnetism, antiferromagnetism, ferrimagnetism

Ordering of the magnetic dipoles in magnetic materials.



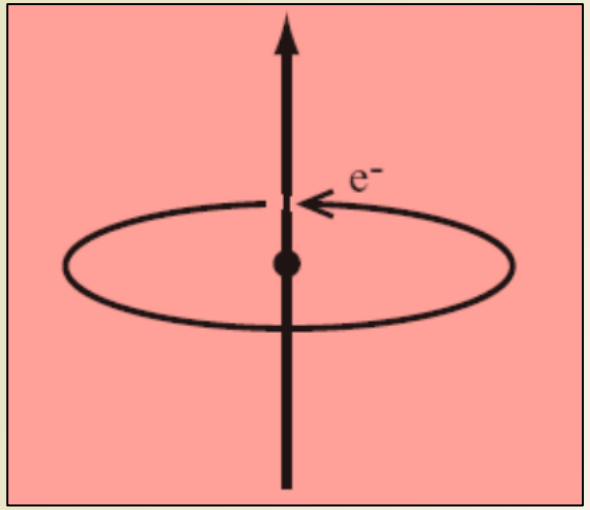
Atomic origins of magnetism

Magnetic moment of a free atom in the absence of a magnetic field consists of *two* contributions.

- **First is due to orbital motion of the electrons circulating the nucleus.**
- **Secondly, each electron has an extra contribution to its magnetic moment arising from its "spin."**

The spin and orbital angular momenta combine to produce the observed magnetic moment.

(The nucleus also has a small magnetic moment, but it is insignificant compared to that of the electrons, hence neglected)



The magnetic moment of an electron, due to orbital motion, may be calculated by an equation: **μ = (area of loop)(current)**

In SI units, the charge of the electron is measured in coulombs. The current, or charge passing a given point per unit time, is then **ev/2πr.**

Therefore, μ (orbit) $= \pi r^2 \left(\frac{evr}{2\pi r}\right) = \frac{evr}{2}$ (SI).

Angular momentum, L = mvr

So, μ =(e/2m) L (Direction of L is opposite to μ)

An additional postulate of the theory was that the angular momentum of the electron must be an integral multiple of h/2π, where h is Planck's constant. Therefore,

$$mvr = nh/2\pi$$

Combining these relations, we have: **μ (orbit) = eh/4πm** (SI unit, for first Bohr orbit, n=1)

The magnetic moment of an electron, due to "Spin"

The spin of the electron was postulated in 1925 in order to explain certain features of the optical spectra of hot gases, particularly gases subjected to a magnetic field (Zeeman effect), and it later found theoretical confirmation in wave mechanics. Spin is a universal property of electrons in all states of matter at all temperatures. The electron behaves as if it were in some sense spinning about its own axis, and associated with this spin are definite amounts of magnetic moment and angular momentum. It is found experimentally and theoretically that the magnetic moment due to electron spin is equal to

$$\mu(\text{spin}) = \frac{eh}{4\pi m} \text{ (SI)}$$
$$= \frac{(1.60 \times 10^{-19}\ \text{C})(6.62 \times 10^{-34}\ \text{J s})}{4\pi(9.11 \times 10^{-31}\ \text{kg})} \text{ (SI)}$$
$$= 9.27 \times 10^{-24}\ \text{J/T or Am}^2.$$

Thus the magnetic moment due to spin and that due to motion in the first Bohr orbit are exactly equal. Because it is such a fundamental quantity, this amount of magnetic moment is given a special symbol μ_B and a special name, the *Bohr magneton*. Thus,

$$\mu_B = \text{Bohr magneton} = eh/4\pi mc = 0.927 \times 10^{-20}\ \text{erg/Oe (cgs)}$$
$$= eh/4\pi m = 9.27 \times 10^{-24}\ \text{A} \cdot \text{m}^2 \text{ (SI)}$$
$$= \mu_0 eh/4\pi m = 1.17 \times 10^{-29}\ \text{Wb} \cdot \text{m (SI)}.$$

μ_B is a natural unit of magnetic moment, just as the electronic charge e is a natural unit of electric charge..

Imagine an electron as a sphere with its charge distributed over its surface. Rotation of this charge produces an array of tiny current loops, each of which has a magnetic moment directed along the rotation axis. But if we calculate the resultant moment of all these loops, we obtain the wrong answer, $(5/6)\mu_B$ instead of μ_B. Nor does the right answer result from the assumption that the charge is uniformly distributed through the volume of the sphere. Such calculations are fruitless, because we do not know the shape of the electron or the way in which charge is distributed on or in it. The spin of the electron, and its associated magnetic moment, has to be accepted as a fact, consistent with wave mechanics and with a large number of experiments of various kinds, but with no basis in classical physics. The model of the following figure is therefore only an aid to visualization; it has no quantitative significance.

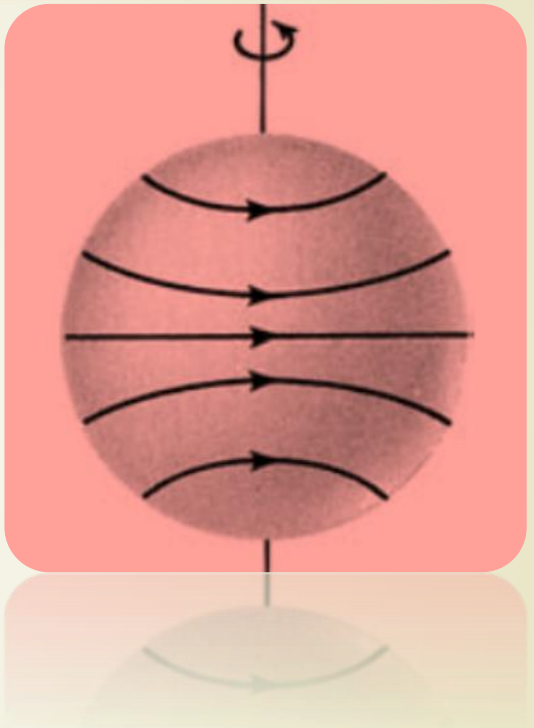


Fig: Visualization of electron spin.

MAGNETIC MOMENTS OF ATOMS

Atoms contain many electrons, each spinning about its own axis and rotates in its own orbit. The magnetic moment associated with each kind of motion is a vector quantity, parallel to the axis of spin and normal to the plane of the orbit, respectively. The magnetic moment of the atom is the vector sum of all its electronic moments, and two possibilities arise:

1. The magnetic moments of all the electrons are so oriented that they cancel one another out, and the atom as a whole has no net magnetic moment. This condition leads to diamagnetism.

2. The cancellation of electronic moments is only partial and the atom is left with a net magnetic moment. Such an atom is often referred to, for brevity, as a magnetic atom. Substances composed of atoms of this kind are para-, ferro-, antiferro-, or ferrimagnetic.

THEORY OF DIAMAGNETISM (*Classical theory of Langevin*)

A diamagnetic is a substance that exhibits negative magnetism. Even though it is composed of atoms which have no net magnetic moment, it reacts in a particular way to an applied field.

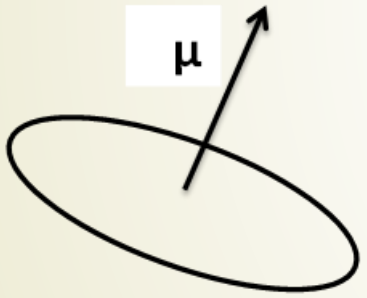
Diamagnetic materials exclude or repel magnetic fields.

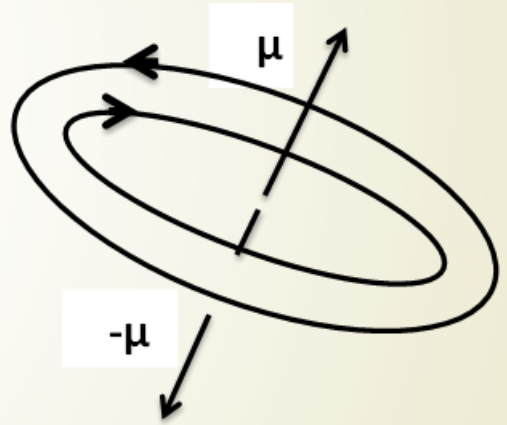
After pouring liquid nitrogen the superconducting material within the bowl becomes diamagnetic and hence repeal the ferromagnetic material.

Magnetic levitation Experiment

The theory considers that the effect of an applied field on a single electron orbit is to reduce the effective current of the orbit, and so to produce a magnetic moment opposing the applied field. This effect is summed over all the electrons in the atom, and each atom is regarded as acting independently of the others. The values of diamagnetic susceptibility calculated in this way generally agree with experimental values to better than a factor of 10, which suggests that the model is at least qualitatively correct. Nothing in the model suggests a strong temperature dependence of susceptibility, and this also agrees with the experiment.

Applied Magnetic Field = 0



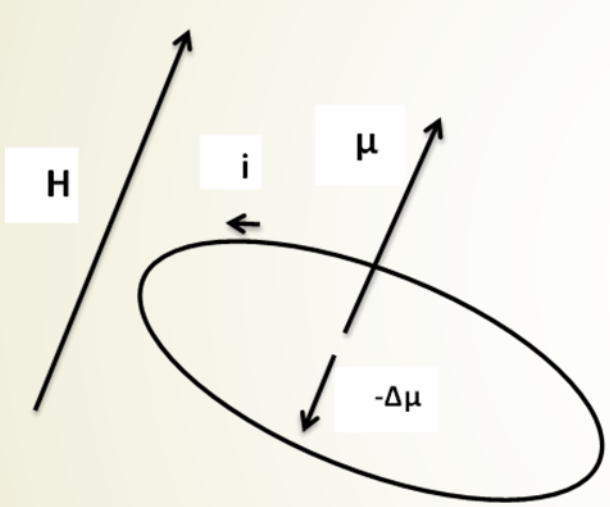


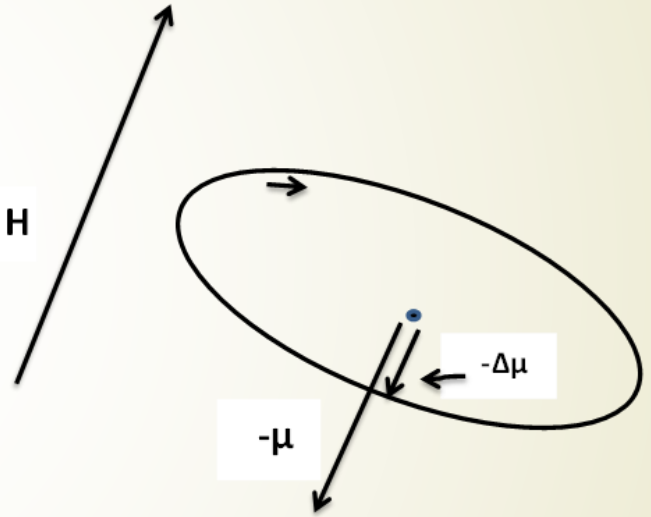
Orbital magnetic moment due to one electron =

$$= \pi r^2 \left(\frac{evr}{2\pi r}\right) = \frac{evr}{2} \text{ (SI).}$$

Net Magnetic Moment for two electrons moving in opposite direction is zero.

Applied Magnetic Field = H





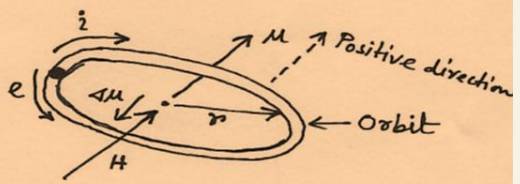
Therefore, net magnetic moment due to these two electrons = -2Δμ

Classical Langevin Theory of Diamagnetism:

A diamagnetic substance exhibits negative magnetization under the application of a magnetic field. However, it is composed of atoms which have no net magnetic moment but it reacts in a particular way to an applied field. The theory was given by Paul Langevin in a classic paper published in 1905 where he worked out the behaviour of diamagnetic substance under the application of field.

Consider an electron orbit normal to an applied field.

Motion of the electron is equivalent to current in a loop, and as soon as the field is increased from zero, the change in flux through the loop induces an emf $\mathcal{E}$ in the loop.



According to Faraday's law $\mathcal{E} = -\frac{d\Phi}{dt}$

$$= -\frac{d}{dt}(BA) \quad ; \text{ where } B = \text{mag. induction} = \mu_0 H$$

$$= -\mu_0 A \frac{dH}{dt} \cdots (1)$$

~~μ_0 = permeability~~

μ_0 = permeability in free space

$= 4\pi \times 10^{-7}$ Weber/Amp. meter

A = Area of the loop.

The negative sign indicates that this emf acts in such a way that opposes the change in flux (Lenz's Law). That is to decrease the flux. This can be achived by a decrease in loop current or in other words by decrease of the electron velocity. So, the result is a decrease in magnetic moment of the loop.

Since the current loop has no resistance, thus the change in current produced by $\mathcal{E}$ persists, even though $\mathcal{E}$ is finite only while the applied field is changing from 0 to H. The effect is not momentary. The mag. moment is decreased as long as the field H is acting.

The electron moving counterclockwise, so the magnetic moment μ of the loop is in the same direction of the applied mag. field. When the magnetic field H applied, it changes μ by an amount $\delta\mu$ in the negative direction. Now imagine another orbit adjacent to that electron, moving in opposite direction. So in the absence of mag. field the net magnetic moment is zero as these two electron moving opposite direction cancel the moment each other. But when a field is applied the first electron produce a change in moment in the negative direction and the second electron also produce a change in the '-ive direction. So, ~~a negative~~ in the presence of a magnetic field a net magnetic moment opposite to the field direction (negative direction) observed.

Now, we shall compute the change in moment of a single orbit as follows:

If E is the electric field intensity acting around a circular orbit of length l, then

$$E = \frac{\mathcal{E}}{l} = -\frac{\mu_0 A}{l}\frac{dH}{dt} \qquad \text{(By using equn 1)}$$

$$= -\mu_0 \frac{\pi r^2}{2\pi r}\frac{dH}{dt}$$

$$= -\frac{\mu_0 r}{2}\frac{dH}{dt}.$$

The force exerted on the electron by this field is

$$F = Ee = ma \qquad (m = \text{mass of an electron})$$

The resultant acceleration is

$$a = \frac{dv}{dt} = \frac{Ee}{m} = -\frac{\mu_0 e r}{2m}\frac{dH}{dt} \quad ---- (2)$$

Integrating over a change in magnetic field from 0 to H, and assuming that the orbital radius r does not change during the application of the field, we have

$$\int_{v_1}^{v_2} dv = -\frac{\mu_0 e r}{2m} \int_0^H dH$$

$$\Rightarrow v_2 - v_1 = \Delta v = \frac{-\mu_0 e r H}{2m}.$$

As we know orbital magnetic moment $\mu = \frac{evr}{2}$

$\therefore$ Change in μ due to change in v

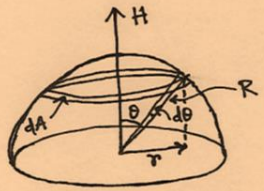
is given by $\Delta\mu = \frac{er(\Delta v)}{2}$

So, the change in magnetic moment $\Delta\mu$

$$\Delta\mu = -\frac{\mu_0 e^2 r^2 H}{4m} \quad \text{------(3)}$$

We obtained this result when the mag. field is perpendicular to the plane of the orbit. In general, it will be inclined to the field, and r in the above equn must then be interpreted as the projection of the orbit radius R on a plane normal to the field.

The figure shows the radius R of an orbit inclined at an angle θ to the field. So, we have to take the average of r^2 when R takes on all possible orientations in the hemisphere.



This average value is

$$\overline{r^2} = \overline{R^2 \sin^2\theta} = \frac{2R^2}{3}.$$

Using this value in equn (3) we get

$\Delta\mu = -\frac{\mu_0 e^2 R^2 H}{6m}$, where 'R' is the radius of an orbit that can take all possible orientations with respect to the field.

So far we have considered only a single electron. The change in moment of an atom containing Z electrons will be given by a sum of Z terms

$$\therefore \quad \Delta\mu \text{ (per atom)} = -\frac{\mu_0 e^2}{6m} H \sum R_n^2$$

where R_n is the radius of the nth orbit.

The summation $\sum R_n^2$ can be replaced by $Z\overline{R^2}$, where $\overline{R^2}$ is the average of the squares of the various orbital radii.

Now, if ρ is the density of the material,
A is the atomic weight
N is the Avogadro number

then number of atoms per unit volume $= \frac{N\rho}{A}$.

Therefore $\Delta\mu$ (per unit vol.) $= -\frac{N\rho}{A}\left(\frac{\mu_0 e^2 Z\overline{R^2}}{6m} H\right)$

$\therefore$ Volume susceptibility is given by $\chi = \frac{M}{H} = \frac{\Delta\mu}{H}$

(since there was no net mag. moment in the absence of field.)

$$\therefore \chi = -\left(\frac{N\rho}{A}\right)\frac{\mu_0 e^2 Z\overline{R^2}}{6m}$$

We derived the susceptibility in SI unit, and it is a dimensionless quantity. We see that the susceptibility is always a negative quantity and there is no explicit temperature dependence term in it, however $\overline{R^2}$ is weakly temperature dependent term.

So, the susceptibility does not varies with temperature, this agrees the experimental observation.

The magnitude of χ obtained from theoretical calculation does not agree so good with that obtained from experiment.

Electrons which constitute a closed shell in an atom usually have their spin and orbital moments oriented so that the atom as a whole has no net moment. Thus the monoatomic rare gases He, Ne, A, etc., which have closed-shell electronic structures, are all diamagnetic. So are most polyatomic gases, such as H_2, N_2, etc., because the process of molecule formation usually leads to filled electron shells and no net magnetic moment per molecule.

VARIATION OF SUSCEPTIBILITY WITH TEMPERATURE

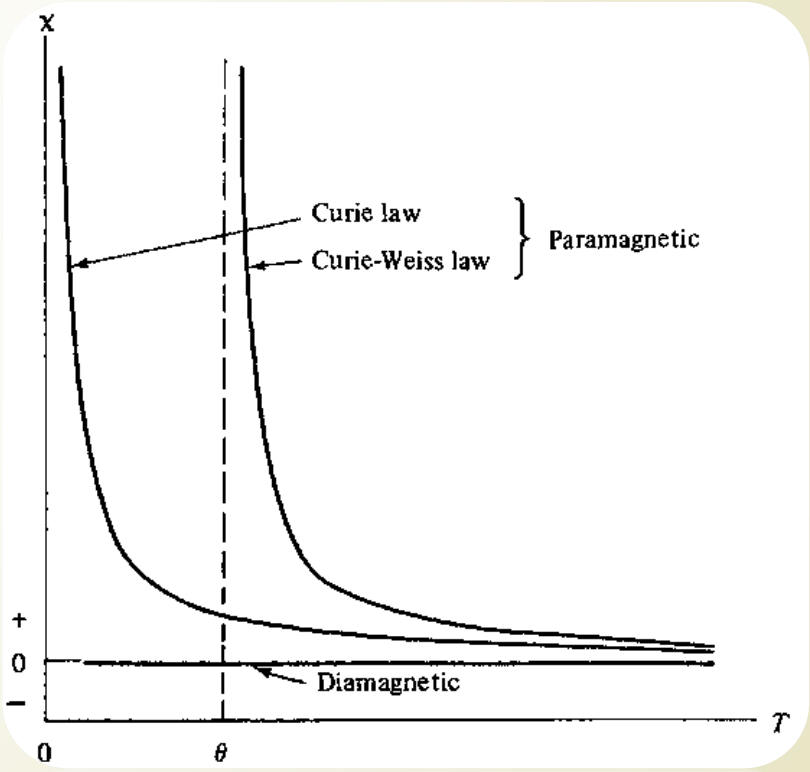
The first systematic measurements of the susceptibility of a large number of substances over an extended range of temperature were done by Pierre Curie and found that the mass susceptibility was independent of temperature for diamagnetics, but it varied inversely with the absolute temperature for paramagnetic materials

$$\chi_m = C/T.$$

This relation is known as ***Curie law***, and C is the Curie constant per gram. It was later shown that the Curie law is only a special case of a more general law, Known as ***Curie –Weiss*** law.

$$\chi_m = C/(T - \theta),$$

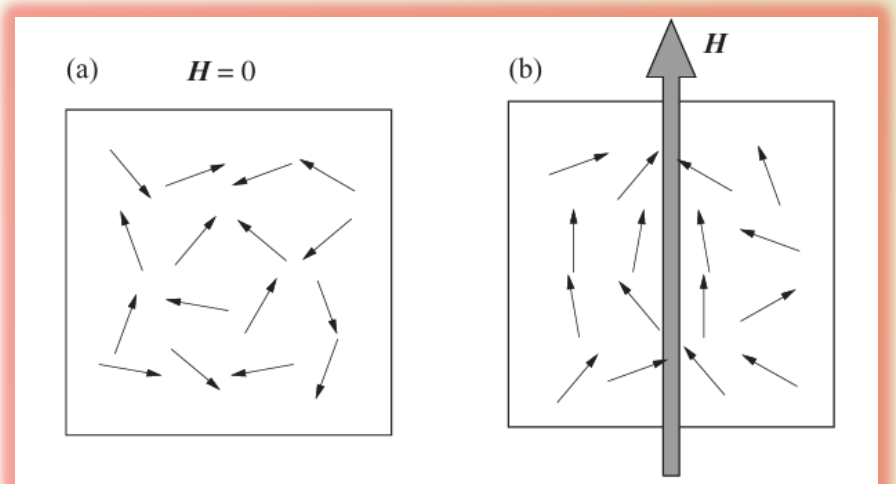
θ is a constant, with dimensions of temperature. C is the Curie constant per gram.



CLASSICAL THEORY OF PARAMAGNETISM (Langevin theory)

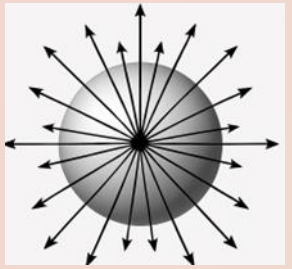
Assumption: **Paramagnetic material consists of atoms, or molecules, each of which has the same net magnetic moment μ, because all the spin and orbital moments of the electrons do not cancel out.**

In the absence of an applied field, these atomic moments point at random and cancel one another, so that the magnetization of the specimen is zero.



When a field is applied, there is a tendency for each atomic moment to turn toward the direction of the field; if no opposing force acts, complete alignment of the atomic moments would be produced and the specimen as a whole would acquire a very large moment in the direction of the field. But thermal agitation of the atoms opposes this tendency and tends to keep the atomic moments pointed at random. The result is only partial alignment in the field direction, and therefore a small positive susceptibility. The effect of an increase in temperature is to increase the randomizing effect of thermal agitation and therefore to decrease the susceptibility.

Consider a unit volume of material containing n atoms, each having a magnetic moment μ. Let the direction of each moment be represented by a vector, and let all the vectors be drawn through the center of a sphere of unit radius.



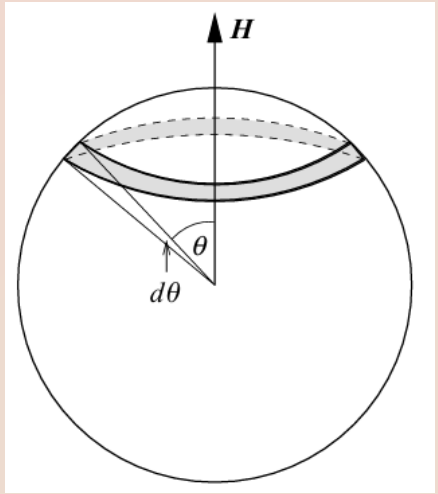
The energy of a magnetic moment in an applied field H is $E_p = -\mu \cdot H = -\mu H \cos\theta$.

Applying Boltzmann statistics the probability that a magnetic moment will be at an angle θ to the applied field is proportional to $\exp(\mu H \cos\theta / KT)$

The number of moments between θ and $\theta + d\theta$ will now be proportional to dA, multiplied by the Boltzmann factor

$$dn = K\, dA\, e^{-E_p/kT} = 2\pi K e^{(\mu H \cos\theta)/kT} \sin\theta\, d\theta,$$

Eq-1



where K is a proportionality factor, determined by $\int_0^n dn = n.$

Putting $a = \mu H/KT$

$$2\pi K \int_0^{\pi} e^{a\cos\theta} \sin\theta\, d\theta = n.$$

Eq--2

The total magnetic moment (M) in the direction of the field acquired by the unit volume (For n atoms)

$$M = \int_0^n \mu \cos\theta \, dn.$$

Substituting Eq-1 and 2 into this expression, we have

$$M = 2\pi K \mu \int_0^{\pi} e^{a\cos\theta} \sin\theta \cos\theta \, d\theta$$

$$= \frac{n\mu \int_0^{\pi} e^{a\cos\theta} \sin\theta \cos\theta \, d\theta}{\int_0^{\pi} e^{a\cos\theta} \sin\theta \, d\theta}$$

To evaluate these integrals, we put x=cosθ and dx= -sinθ dθ then

$$M = \frac{n\mu \int_1^{-1} x e^{ax} \, dx}{\int_1^{-1} e^{ax} \, dx}$$

$$= n\mu \left(\frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} \right) = n\mu \left(\coth a - \frac{1}{a} \right).$$

$$\frac{M}{M_0} = \coth a - \frac{1}{a}$$ =L(a),

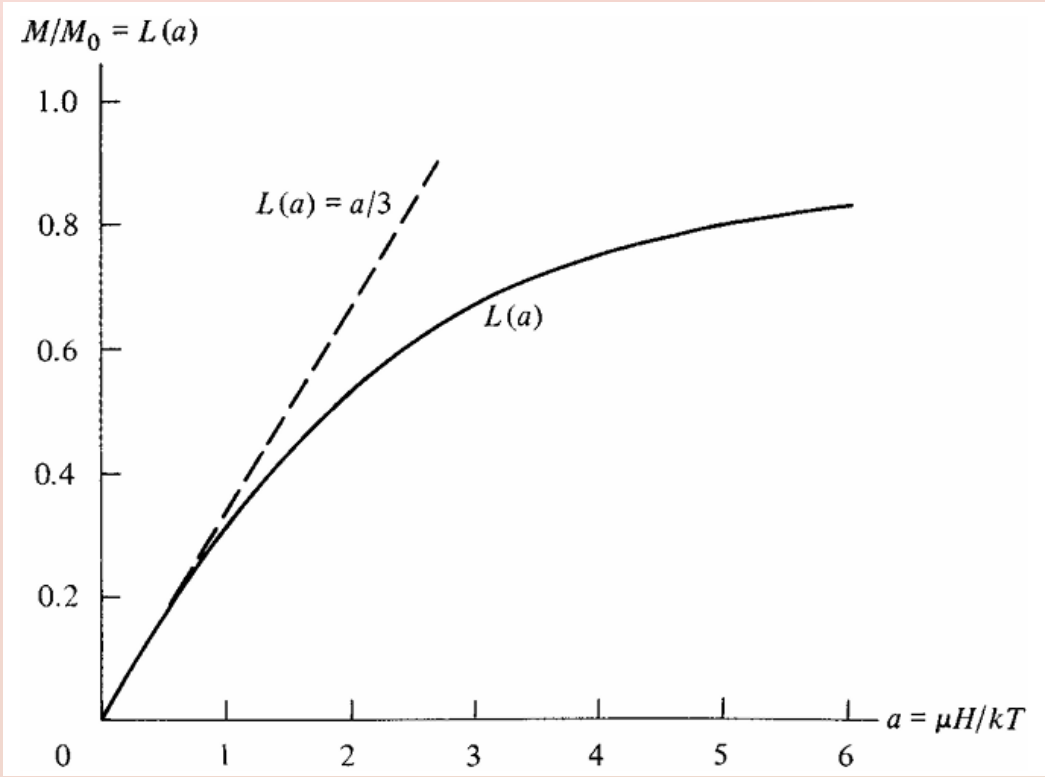
$M_0 = n\mu$ = Saturation Magnetization

$$L(a) = \frac{a}{3} - \frac{a^3}{45} + \frac{2a^5}{945} - \cdots$$

The Langevin theory leads to two conclusions:

1. Saturation will occur if a (=μH/kT) is large enough. This makes good physical sense, because large H or low T, or both, is necessary if the aligning tendency of the field is going to overcome the disordering effect of thermal agitation.

2. At small a, the magnetization M varies linearly with H. As we shall see presently, a is small under "normal" conditions, and linear M, H curves are observed



Langevin function

The Langevin theory leads to the Curie law. For small a, L(a) =a/3, we get the following:

Where ρ is density. The number of atoms per unit volume (n), is equal to Nρ/A, where N is atoms/mol (Avogadro's number), and A is atomic weight.

$$M = \frac{n\mu a}{3} = \frac{n\mu^2 H}{3kT}.$$

$$\chi_v = \frac{M}{H} = \frac{n\mu^2}{3kT},$$

$$\chi_m = \frac{\chi_v}{\rho} = \frac{n\mu^2}{3\rho kT}.$$

$$\chi_v = \frac{N\mu^2}{3AkT} = \frac{C}{T}\ \frac{\text{emu}}{\text{cm}^3\ \text{Oe}}\ (\text{cgs})\ \text{or}\ \frac{\text{Am}^2}{\text{m}^3\,\text{Am}^{-1}}\ [\text{dimensionless}](\text{SI})$$

$$\chi_m = \frac{N\mu^2}{\rho 3AkT} = \frac{C}{\rho T}\ \frac{\text{emu}}{\text{g Oe}}\ (\text{cgs})\ \text{or}\ \frac{\text{Am}^2}{\text{kg Am}^{-1}} = \frac{\text{m}^3}{\text{kg}}\ (\text{SI})$$

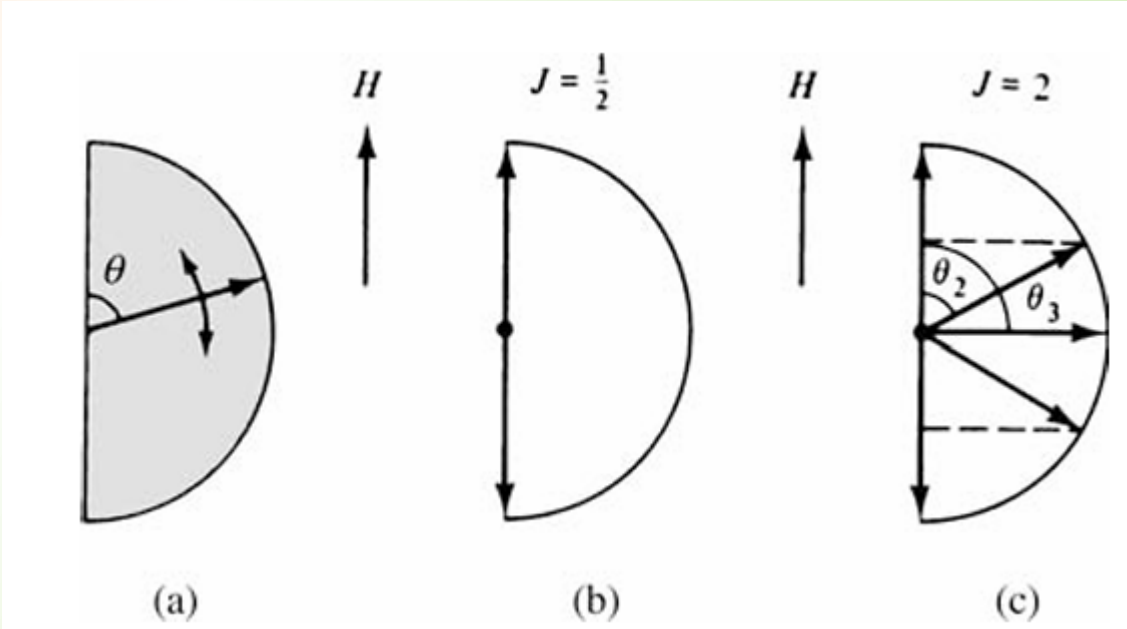
Where C is the Curie constant, given by $\mathbf{C} = \frac{N\mu^2}{3AK}$

QUANTUM THEORY OF PARAMAGNETISM

Atomic magnetic moment (μ) makes angle (θ) with the applied magnetic field (H) can take some discrete values instead of continuous value. This restriction is called space quantization.

The possible values of μ_H, the component of μ_{eff} in the direction of the applied field H. These possible values are $\mu_H = gM_J\mu_B$.

Where M_J is a quantum number associated with J. For an atom with a total angular momentum J, the allowed values of M_J are: J, J -1, J-2,..., -(J-2), -(J-1), -J



Space quantization: (a) classical, (b) and (c), two quantum possibilities.

The potential energy of each moment in the field **H** is $\mathbf{E_p = -gM_J\mu_B H}$

If there are **n** atoms per unit volume, the magnetization **M** is given by the product of **n** and the average magnetic moment resolved in the direction of the field

$$M = n\frac{\sum gM_J\mu_B e^{gM_J\mu_B H/kT}}{\sum e^{gM_J\mu_B H/kT}},$$

$$M = ngJ\mu_B\left[\frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}\right)a' - \frac{1}{2J}\coth\frac{a'}{2J}\right],$$

$$a' = \frac{gJ\mu_B H}{kT} = \frac{\mu_H H}{kT}.$$

$$M = n(\mu_{\mathrm{H}})_{\mathrm{av}},$$

$$M = n\mu_{\mathrm{B}} \frac{e^{\mu_{\mathrm{B}}H/kT} - e^{-\mu_{\mathrm{B}}H/kT}}{e^{\mu_{\mathrm{B}}H/kT} + e^{-\mu_{\mathrm{B}}H/kT}},$$

$$M = M_0 \tanh \frac{\mu_{\mathrm{B}}H}{kT},$$

$$\frac{M}{M_0} = \tanh a'.$$

(classical) $a = \mu H/kT,$

(quantum) $a' = \mu_{\mathrm{H}}H/kT.$

$$
\begin{aligned}
M &= ngJ\mu_{\mathrm{B}}B(J, a') \\
&= ngJ\mu_{\mathrm{B}}\left(\frac{gJ\mu_{\mathrm{B}}H}{kT}\right)\left(\frac{J+1}{3J}\right) \\
&= \frac{ng^2J(J+1)\mu_{\mathrm{B}}^2H}{3kT} \\
&= \frac{n\mu_{\mathrm{eff}}^2H}{3kT}.
\end{aligned}
$$

The mass susceptibility is then

$$
\chi_{\mathrm{m}} = \frac{\chi_v}{\rho} = \frac{M}{\rho H} = \frac{n\mu_{\mathrm{eff}}^2}{3\rho kT} = \frac{N\mu_{\mathrm{eff}}^2}{3AkT}.
$$

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