2021

PHYSICS—HONOURS

Paper : CC-5

Full Marks : 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any four*** from the rest.

1. Answer ***any five*** questions: 2×5

(a) Find the average value of $sin\, x + \sin^2 x$ for $0 \leq x \leq 2\pi$.

(b) Using generating function, show that $p_l(-x) = (-1)^l p_l(x)$, where $P_l(x)$ is Legendre polynomial of order *l*.

(c) Find the solution of the equation $x^2 \frac{d^2y}{dx^2} + px\frac{dy}{dx} + qy = o$, where p and q are constants.

(d) For the Poisson's distribution $P_n = \frac{\mu^n}{n!} e^{-\mu}$. Find the standard deviation for the distribution. μ is constant.

Or, [syllabus 2018-2019]

If the Lagrangian is invariant under a rigid translation, the momentum of the system is conserved.

(e) If $f(x) \to 0$ for $x \to \pm\infty$, find the Fourier transform of $\frac{df}{dx}$.

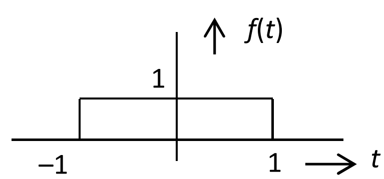
Or, [syllabus 2018-2019]

Show that if the Hamiltonian is not explicitly time dependent then energy is conserved (assuming Hamiltonian equals energy).

(f) Show that $x = 1$ is a regular singular point of the Legendre differential equation.

(g) Show that, for any $p > 0$, $\Gamma(p+1) = p\Gamma(p)$.

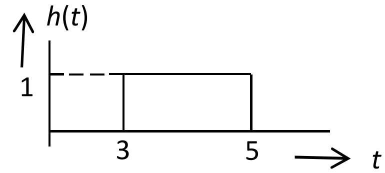
2. Consider the function shown in figure



(a) Find its Fourier transform g(ω). Sketch g(ω)vs ω.

(b) Show, using Parseval's identity $\int_0^\infty \frac{\sin^2 \omega}{\omega^2} d\omega = \frac{\pi}{2}$.

(c) Show that Fourier transform of $f(t - t_0) = e^{-i\omega t_0}\, g(\omega)$.

(d) 

Using (a) and (c), find Fourier transform of $h(t)$. (2+1)+3+2+2

Or, [Syllabus 2018-2019]

(a) Find the path followed by a light ray if the index of refraction in polar coordinate is proportional to r^{-2}.

(b) Find the equation of motion of a particle moving along x–axis under a potential energy $V = \frac{1}{2}kx^2$, by constructing the Lagrangian. Construct the Hamiltonian for the system and argue that it is a conservative system. 5+5

3. (a) Calculate, using gamma function $\int_1^\infty \frac{(lnx)^3}{x^2(x-1)}\, dx$ $\left(\text{you can assume } \sum_{r=1}^{\infty} \frac{1}{r^4} = \frac{\pi^4}{90}\right)$.

(b) Prove that for positive integers m and n, $\beta(m,n) = \frac{(n-1)!}{m.(m+1)...(m+n-1)}$.

Hence show that 1.3.5 ... $(2n-1) = \frac{2^n\, \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}}$. 5+(3+2)

4. (a) Given $f(x) = x$ for $0 < x < 1$, sketch even function corresponding to this function with period 2. Find the Fourier series for this even function.

(b) Using above expansion, find the value of $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

(c) Sketch the odd function of period 2 corresponding to the above given function. 5+3+2

5. (a) From the generating function of Hermite polynomial $H_n(x)$, $e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x)$

Show that $H_n'(x) = 2n\, H_{n-1}(x)$.

(b) $H_n(x)$ are orthogonal polynomials in the domain $-\infty < x < \infty$. Supose $H_2(x) = a + bx^2$. Find a and b using orthogonality property of $H_n(x)$; given $H_0(x) = 1$, $H_1(x) = 2x$ and coefficient of highest power of x in $H_n(x)$ is 2^n.

(c) Solve $x^2y'' - 6y = 0$ using Frobenius method around $x = 0$. 2+4+4

6. (a) For a binomial distribution with n trials, if p is the probability of success and q is that of failure, then show that the mean and variance of the distribution are respectively np and npq.

(b) Solve $\frac{\partial^2 y}{\partial x^2} = c\frac{\partial y}{\partial t}$ using Fourier transform.

(c) A random variable x has the density function $f(x) = \begin{cases} cx & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$

find (i) the constant c.

(ii) the probability that $x > 1$. (2+2)+3+(1+2)

Or, [Syllabus 2018-2019]

(a) Derive the Euler-Lagrange equation from the Principle of Least Action. Show clearly how the variation of paths is implemented in your derivation in terms of ordinary partial derivative with respect to a parameter.

(b) A bead slides on a frictionless wire in the shape of a cycloid described by the equations

$$x = a(\theta - sin\theta) \qquad y = a(1 + cos\theta)$$

where (x, y) are cartesian coordinates, a is a constant and $0 \le \theta \le 2\pi$. Write down (i) the Lagrangian and (ii) the equation of motion in terms of θ. Indicate the generalized coordinate θ by drawing a figure. 5+(2+2+1)

7. (a) The heat equation in 2 dimensional cartesian coordinates is given by $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{K}\frac{\partial T}{\partial t}$

where T is the temperature function and K is a constant. Solve this equation for steady state using the method of separation of variables.

(b) Find a solution $U(x, t)$ of the boundary-value problem $\frac{\partial U}{\partial t} = 3\frac{\partial^2 U}{\partial x^2}$ $t>0,\ 0<x<2$

The boundary conditions are

$U(0,t) = 0,\ U(2,t) = 0$

$U(x,0) = x \quad 0 < x < 2$

$\left[\text{Given } x = \sum_{n=1}^{\infty} -\frac{4}{n\pi}(1)^n \sin\frac{n\pi x}{2}; 0 < x < 2\right]$ 3+7